

Space-like meson electromagnetic form factor in a relativistic quark model

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The space-like electromagnetic form factor is expressed in terms of the overlap integral of the initial and final meson wave functions written as Lorentz covariant distributions of internal momenta. The meson constituents are assumed to be valence quarks and an effective vacuum-like field. The momentum of the latter represents a relativistic generalization of the potential energy of the quark system.

The calculation is fully Lorentz covariant and the form factors of the charged mesons are normalized to unity at $t=0$. The numerical results have been obtained by freezing the transverse degrees of freedom. We found $r_\pi^2=0.434 \text{ fm}^2$, $r_{K^+}^2=0.333 \text{ fm}^2$, $r_{K^0}^2=-0.069 \text{ fm}^2$ by taking $m_{u,d}=430 \text{ MeV}$, $m_s=700 \text{ MeV}$.

Key words: electromagnetic form factors; quark models

I. INTRODUCTION

In the standard model and the lowest order of perturbation the elastic electron meson scattering is the result of one photon exchange between the electron and one of the quarks in the meson. Formally the hadronic matrix element entering the expression of the scattering amplitude is parametrized in a model independent Lorentz covariant way as:

$$\langle M(P') | U_s(+\infty, 0) J_{em}^\mu(0) U_s(0, -\infty) | M(P) \rangle = T^\mu = f_{em}(t) (P + P')^\mu \quad (1)$$

where J_{em}^μ is the elementary electromagnetic current expressed in terms of free quark fields, $U_s(\tau, \tau')$ is the time evolution operator of the system due to quantum fluctuations of the colour field and $t = (P - P')^2$ is a space-like momentum transfer. All the information concerning the quark structure of the meson and the effects of the quantum fluctuations of the coloured field is contained in the electromagnetic form factor $f_{em}(t)$. This makes the calculation of the electromagnetic form factor a rather complicate problem.

As shown in a careful analysis [1], perturbative QCD calculations solely cannot explain the observed pion and nucleon form factors at presently accessible values of t [2]. The dominant contribution seems to come from the hadronic wave functions which are essentially nonperturbative.

In fact this was to be expected from the uncertainty principle of quantum mechanics. Point-like quarks and a real local interaction between them could be observed only asymptotically. At low momentum

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transfer only dressed quarks, the so-called constituent quarks, are observable. Moreover, the electromagnetic current expressed in terms of constituent quark fields is only approximately local and the binding may also induce sizable effects.

In the present paper we calculate the form factors at low momentum transfer, where mesons exhibit a stable, time invariant structure in terms of constituent quarks. We assume that, to a good extent, the electromagnetic current expressed as a product of constituent quark fields may be considered local. Consequently the matrix element (1) will be related to the overlap integral between the wave function of the final meson and that of the initial meson where one of the quarks has been replaced by the quark interacting with the electromagnetic field. We therefore consider that the space-like form factor indicates to what extent the initial system, where one of the quarks has been replaced by the recoiling quark, represents the final meson. Similar significance can be assigned also to the semileptonic form factors.

In this framework, at low space-like t the form factor $f_{em}(t)$ is the effect of a stable meson structure [1] and one would expect it to decrease continuously with $|t|$ because the larger is the momentum transfer, the less probable is to find the recoiling quark in the final meson. The decrease is then nothing else but the consequence of the worse and worse matching of the initial with final wave functions.

Then the calculation of the form factors at low momentum transfer amounts in finding a Lorentz covariant internal wave function for the meson as a bound system made out of independent constituents.

One of the best known solution to this problem is the Bethe Salpeter wave function. Completed with the expression of the dressed quark-photon vertex satisfying the Dyson-Schwinger equation it led to remarkable results for low energy observables [3], [4]. It must be noticed however that the formalism has ambiguities related to the definition of the relative quark momentum and that the presence of negative energy states in the quark propagators violates the real quark content of mesons. Also, in order to satisfy the usual normalization condition of the form factor additional assumptions seem to be necessary [1], [4].

Quark models also used solutions of the Schrödinger equation with suitably chosen potential terms. The calculations have been performed at $t = 0$ and the decrease with $|t|$ has been introduced from phenomenological reasons [5].

Results independent of the specific form of the confining potential have been obtained in the light-front approach [6] where the potential is formally included into the free mass term of the quark-antiquark system. The form factors can be calculated at any t , but manifest Lorentz covariance is lost because of the difference between "good" and "bad" components of the current.

In this paper we present an original way to take into account the confinement [7]. The specific assumption of the model is that at low energy the hadrons look like stable systems made of valence quarks *and* of an effective vacuum-like field Φ representing countless excitations of the quark-gluonic field which cannot be observed one by one. The sum of all their momenta is the 4-momentum of the field Φ which is supposed to be the relativistic generalization of the potential energy of the quark system.

The main reason for introducing this effective description is suggested by the examples taken from relativistic field theory, where a bound state is the result of an infinite series of elementary interactions, but not the instantaneous effect of an elementary process [3].

We therefore conjecture that the binding forces are the result of a time average of elementary processes which cannot be seen one by one if the time of observation is rather long (i.e. if the momentum transfer from the projectile to the target is rather low). We consider that the countless elementary excitations continuously taking place in a bound system deserve an average description in terms of an effective field Φ , not an individual, perturbative treatment. Furthermore, the presence besides the valence quarks of an effective field Φ whose 4-momentum does not satisfy any mass constraint may also be useful for ensuring a definite mass to the compound system. Indeed, as it is well known, a system made of on-mass-shell particles having a continuous distribution of relative momenta does not behave like a single particle because it does not have a definite mass [5]. An effective field may improve the situation by playing the role of a potential whose contribution adds to the quark kinetic energy yielding a definite value for the meson mass.

The main features of the model are presented in the next section. We give the generic form of the meson state and calculate the expression of the electromagnetic form factor by using the canonical formalism of field theory.

The third section contains the numerical results obtained with frozen transverse degrees of freedom.

In the last section we draw some conclusions concerning the applicability of the model and comment on the values of the parameters yielding the best fit.

II. THE MODEL

The main feature of the model is to suggest the generic wave function of the bound quark system representing the meson. A first requirement for the wave function is to be a Lorentz covariant function of the independent quark coordinates because the quarks really behave like independent particles in the interaction with the electromagnetic field. It is obvious that in order to satisfy this requirement without introducing unphysical coordinates it is necessary to work in momentum space. This will also help expressing easily the mass shell constraints and the conservation laws. For this reason the internal wave function of the meson will be given under the form of the distribution function of the internal momenta.

Another constraint for the meson wave function follows from the requirement to satisfy a single particle normalization condition. Accordingly, the integral over the internal momenta must converge, the relative momentum cut-off being provided by the internal distribution function of the quark momenta.

Recalling our conjecture that the binding potential can be described with the aid of an effective field we assume that the generic form of a single meson state is [7]:

$$|M_i(P)\rangle \int d^3p \frac{m_1}{e_p} d^3q \frac{m_2}{e_q} d^4Q \times \delta^{(4)}(p+q+Q-P) \varphi(p,q;Q) \times \bar{u}(p)\Gamma_M v(q) \chi^+ \lambda_i \psi \Phi^\dagger(Q) a^\dagger(p)b^\dagger(q)|0\rangle \quad (2)$$

where a^+, b^+ are the creation operators of the valence $q\bar{q}$ pair; u, v are Dirac spinors and Γ_M is a Dirac matrix ensuring the relativistic coupling of the quark spins. In the case of pseudoscalar mesons $\Gamma_M = i\gamma_5$. The quark creation and annihilation operators satisfy canonical commutation relations and commute with $\Phi^+(Q)$, which represents the mean result of the elementary quark-gluon excitations responsible for the binding. Their total momentum Q_μ is not subject to any mass shell constraint and, in some sense, it is just what one needs to be added to the quark momenta in order to obtain the real meson momentum. This is in agreement with our assumption that Q is the relativistic generalization of the potential energy but we shall avoid for a while making any definite assumptions about the momentum carried by the effective field.

The internal function of the meson is the Lorentz invariant momentum distribution function $\varphi(p,q;Q)$ which is supposed to be time independent, because it describes an equilibrium situation. This means that it does not change under the action of internal strong forces and hence the time evolution operators $U_s(\tau, \tau')$ in eq. (1) can be replaced by unity. As mentioned above, the main rôle of φ is to ensure the single particle behaviour of the whole system, by cutting off the large relative momenta.

In the evaluation of the matrix element (1) we shall use the canonical commutation relations of the quark operators

$$\{a_i(k), a_j^\dagger(q)\} = \{b_i(k), b_j^\dagger(q)\} = (2\pi)^3 \frac{e_k}{m} \delta_{ij} \delta^{(3)}(k-q) \quad (3)$$

and the expression of the vacuum expectation value of the effective field which is defined as follows [7]:

$$\langle 0 | \Phi(Q_1) \Phi^+(Q_2) | 0 \rangle = (VT_0)^{-1} \int d^4X e^{i(Q_2-Q_1)_\mu X^\mu} = (2\pi)^4 (VT_0)^{-1} \delta^{(4)}(Q_1-Q_2) \quad (4)$$

where V is the meson volume and T_0 is the characteristic time involved in the definition of the mean field Φ . T_0 is the time needed by the bound state to be formed and hence we expect it to be of the order of the hadronization time.

It is important to remark that the definition (4) is compatible with the norm of the vacuum state if one takes $\Phi(0) = 1$. We notice also that the relation (4) has the character of a conservation law, just like the commutation relations (3), both of them being necessary for the fulfillment of the overall energy momentum conservation in the process.

As a first test of the model we evaluate the norm of the single meson state (2) according to the usual procedure. The exponent in the integral $\int_T dX_0 e^{i(E(P)-E(P'))X_0}$ coming from the $\delta^{(4)}$ functions in eqs. (2) and (4) shall be put equal to 0 and the integral equal to T , because the uncertainty in the meson mass is expected to be much smaller than $1/T$.

Observing that T is the characteristic time involved in the definition of the effective field Φ in the case of a moving meson, we write it as $T = \frac{E}{M} T_0$ and get:

$$\langle \mathcal{M}(P') | \mathcal{M}(P) \rangle = 2E (2\pi)^3 \delta^{(3)}(P - P') \mathcal{J} \quad (5)$$

where

$$\mathcal{J} = \frac{-1}{2MV} \int d^3p \frac{m_1}{e_p} d^3q \frac{m_2}{e_q} d^4Q \delta^{(4)}(p + q + Q - P) |\varphi(p, q; Q)|^2 Tr \left(\frac{\hat{p} + m_1}{2m_1} \gamma_5 \frac{\hat{q} - m_2}{2m_2} \gamma_5 \right) = 1. \quad (6)$$

This is a remarkable result because it shows that the wave function of the many particle system representing the meson can be normalized like that of a single particle if the integral \mathcal{J} converges.

As a matter of consistency, we also remark the disappearance of the rather arbitrary time constant T_0 from the expression (5) of the norm.

We evaluate now the matrix element (1) proceeding in the same manner as before. The expression of the electromagnetic current written in terms of free quark fields is

$$J_{em}^\mu(x) = \frac{1}{(2\pi)^3} \sum_i \kappa_i \bar{\psi}_i(x) \gamma^\mu \psi_i(x) \quad (7)$$

where κ_i is the fraction of the proton charge carried by the quark i . Introducing (7) between the meson states (2) and using the relations (4) and (3) to eliminate some integrals over the internal momenta we obtain after a straightforward calculation:

$$\mathcal{T}_\mu = \mathcal{T}_\mu^{(1)} + \mathcal{T}_\mu^{(2)} = -\frac{1}{VT} \int d^4Q \frac{d^3p}{2e_p} \frac{d^3q}{2e_q} \frac{d^3k}{2e_k} \delta^{(4)}(p + q + Q - P) \varphi_i(p, q; Q) (t_\mu^{(1)} + t_\mu^{(2)}) \quad (8)$$

where

$$t_\mu^{(1)} = \kappa_1 \delta^{(4)}(k + q + Q - P') \varphi_f^{(1)}(k, q; Q) Tr \left[\gamma_5 (\hat{k} + m_1) \gamma_\mu (\hat{p} + m_1) \gamma_5 (-\hat{q} + m_2) \right] \quad (9)$$

$$t_\mu^{(2)} = \kappa_2 \delta^{(4)}(p + k + Q - P') \varphi_f^{(2)}(p, k; Q) Tr \left[\gamma_5 (-\hat{k} + m_2) \gamma_\mu (-\hat{q} + m_2) \gamma_5 (\hat{p} + m_1) \right]. \quad (10)$$

The two terms in (8) represent the contributions of the valence quarks, k is the momentum of the recoiling quark, P' is the final meson momentum and $\varphi_{i,f}$ are the momentum distribution functions of the initial and final mesons respectively.

In the following we shall work in the Breit frame where the momenta of the initial and final mesons are $P = (E, 0, 0, P)$ and $P' = (E, 0, 0, -P)$ and the electromagnetic form factor reads as:

$$f_{em}(t) = \frac{1}{\sqrt{4M^2 - t}} \mathcal{T}_0. \quad (11)$$

In this frame it is an easy matter to show that $f_{em}(0)=1$ in the case of a charged meson. The demonstration makes use of $\delta^{(3)}(\vec{k} + \vec{q} + \vec{Q})$ to eliminate the integrals over \vec{k} in $\mathcal{T}_0^{(1)}$ and of the identity

$(\hat{p} + m)\gamma_0(\hat{p} + m) = 2e_p (\hat{p} + m)$ to reduce the number of projectors. Performing a similar operation on $\mathcal{T}_0^{(2)}$ and proceeding like in the case of the norm, one gets

$$\mathcal{T}_0 = 2M (\kappa_1 - \kappa_2)\mathcal{J} \quad (12)$$

which means

$$f_{em}(0) = 1 \quad (13)$$

if the meson wave function is properly normalized.

In order to calculate the form factor at $t \neq 0$ we start by using the $\delta^{(3)}$ functions to eliminate the integrals over the momenta \vec{q} and \vec{k} in the expression of $\mathcal{T}_\mu^{(1)}$ and over \vec{p} and \vec{k} in the expression of $\mathcal{T}_\mu^{(2)}$. After performing the traces over γ matrices we get

$$\begin{aligned} \mathcal{T}_\mu^{(1)} = & \frac{\kappa_1}{VT} \int de_p dp_z d\phi_p d^4Q \frac{1}{4e_k e_q} \delta(e_p + e_q + Q_0 - E) \delta(e_p - e_k) \\ & \times \varphi_i(p, q; Q) \varphi_f^{(1)}(k, q; Q) \{q_\mu t + 2\vec{P} \cdot \vec{Q} (k_\mu - p_\mu) + (k_\mu + p_\mu)[\mathcal{E} + \frac{1}{2}t - 2m_1(m_1 + m_2)]\} \end{aligned} \quad (14)$$

where

$$\mathcal{E} = (E - Q_0)^2 + \vec{P}^2 - \vec{Q}^2 + m_1^2 - m_2^2. \quad (15)$$

Next, by writing

$$\frac{1}{2e_k} \delta(e_k - e_p) = \frac{1}{4P} \delta(p_z - P) \quad (16)$$

and

$$\frac{1}{2e_q} \delta(e_p + e_q + Q_0 - E) = \frac{1}{2p_T Q_T} \delta\left(\cos\phi_p - \frac{\mathcal{E} - 2e_p(E - Q_0)}{2p_T Q_T}\right) \quad (17)$$

we perform the integrals over p_z and ϕ_p in eq. (14).

Then $\mathcal{T}_0^{(1)}$ becomes:

$$\begin{aligned} \mathcal{T}_0^{(1)} = & \frac{\kappa_1}{4VTP} \int d^4Q \int_{e_{pm}}^{e_{pM}} de_p \varphi_i(p, q; Q) \varphi_f^{(1)}(k, q; Q) \frac{1}{2p_T Q_T \sqrt{1 - \cos^2\phi_p}} \\ & \times \{2e_p [\mathcal{E} - 2m_1(m_1 + m_2)] + (E - Q_0) t\}. \end{aligned} \quad (18)$$

The integration limits over e_p result from the kinematical constraints $e_p^2 \geq m_1^2 + P^2$ and $\cos^2\phi_p \leq 1$ which give:

$$\begin{aligned} e_{pM} &= \frac{(E - Q_0)\mathcal{E} + Q_T \sqrt{\mathcal{E}^2 - 4[(E - Q_0)^2 - \vec{Q}_T^2](m_1^2 + \vec{P}^2)}}{2[(E - Q_0)^2 - \vec{Q}_T^2]} \\ e_{pm} &= \frac{(E - Q_0)\mathcal{E} - Q_T \sqrt{\mathcal{E}^2 - 4[(E - Q_0)^2 - \vec{Q}_T^2](m_1^2 + \vec{P}^2)}}{2[(E - Q_0)^2 - \vec{Q}_T^2]}. \end{aligned} \quad (19)$$

The term $\mathcal{T}_0^{(2)}$ can be processed in the same manner, giving a similar expression.

Using the above results it is possible to calculate the electromagnetic form factors at any any t , by choosing an appropriate function φ . In principle, the calculation does not imply any other approximation,

but it is hard to believe that the multiple integral entering the expression of the form factor can be performed exactly.

The expression (18) is, of course, valid for $t \neq 0$, but the infinite value one gets in the limit $t \rightarrow 0$ seems to contradict the normalization of the electric charge (13) which has been demonstrated previously.

This is a disturbing situation which deserves a careful examination. Looking back, we remark that the contradiction comes from the evaluation of some δ functions:

$$\delta(\vec{p} + \vec{q} + \vec{Q} - \vec{P})\delta(e_p + e_q + Q_0 - E(P))\delta(\vec{k} + \vec{q} + \vec{Q}' - \vec{P}')\delta(e_k + e_q + Q_0 - E(P'))\delta^{(4)}(Q - Q') \quad (20)$$

which have been written as

$$\delta(\vec{p} + \vec{q} + \vec{Q} - \vec{P})\delta(e_p + e_q + Q_0 - E(P))\delta(\vec{p} - \vec{k} - 2\vec{P})\delta(e_p - e_k)\delta^{(3)}(\vec{Q} - \vec{Q}')\delta(Q_0 - Q'_0) \quad (21)$$

at $t \neq 0$, while at $t=0$ they have been written as

$$\delta(\vec{p} + \vec{q} + \vec{Q})\delta(e_p + e_q + Q_0 - M) \times \delta(\vec{p} - \vec{k})\delta(Q_0 - Q'_0 - M + M')\delta^{(3)}(\vec{Q} - \vec{Q}')\frac{1}{2\pi} \int e^{i(M-M')X_0} dX_0. \quad (22)$$

In the last expression the integral has been replaced by T because it was assumed that the uncertainty in the meson mass is much smaller than T^{-1} . As it was shown above, at $t = 0$ the magnitude of T does not really matter because it disappears from the expression of the norm. This is not the case at $t \neq 0$ where T remains in the final expression of the form factor. Searching for a way to cure the disagreement between the two cases we conjecture that T must be seen as the overlapping time of the initial and final systems. Of course, T depends on the reference system we consider. On the other hand, the form factor is scalar under Lorentz transformations which means that we have to choose a particular reference frame and write T in terms of Lorentz invariant quantities. (We recall that a similar problem occurs in the definition of the scattering cross-sections, where the flux of the incident particles is written in terms of the relative momentum in the center of mass system.) We choose the Breit frame and consider that T is the time necessary for an object of length L to pass along another object of length L . We put then $T = \frac{L}{E}$ where $L = L_0 \frac{M}{E}$ is the Lorentz contracted length of the meson box of size L_0 in its rest frame.

The numerical results quoted in this paper have been obtained by replacing the symmetry scheme based on the full Lorentz group with the symmetry under the collinear group which is equivalent with the flux tube model with frozen transverse degrees. In this respect our working approximation may be considered as opposite to the light front approach [6] where the transverse degrees of freedom are explicitly taken into account, while the confining potential is assumed to give a fixed contribution to the free meson mass.

In the present approach the relativistic generalization of the potential energy of the quark system is explicitly taken into account as the 4-momentum of an effective field and the mean contribution of the transverse degrees of freedom to the meson energy is included in the quark masses. The longitudinal and the temporal degrees of freedom are then the only active and the multiple integral in eq.(18) reduces to a simple one.

In this case the expressions of $\mathcal{T}_0^{(1)}$ becomes:

$$\begin{aligned} \mathcal{T}_0^{(1)} = & \frac{2\kappa_1}{VT} \int dQ_0 dQ_z \frac{dp_z}{2e_p} \frac{dq_z}{2e_q} \frac{dk_z}{2e_k} \delta^{(2)}(p + q + Q - P)\delta^{(2)}(k + q + Q - P') \\ & \times \varphi_i(p, q; Q)\varphi_f^{(1)}(k, q; Q) \{ (e_p + e_k) [(E - Q_0)^2 - Q_z^2 - P^2 - (m_1 - m_2)^2] + e_q t \} \end{aligned} \quad (23)$$

Proceeding in the same manner as in the three dimensional case we get:

$$\begin{aligned} \frac{1}{2e_k} &= \frac{1}{4P}\delta(p_z - P) \\ \frac{1}{2e_q}\delta(e_p + e_q + Q_0 - E) &= \delta(m_2^2 + Q_z^2 - (E - Q_0 - e_p)^2). \end{aligned} \quad (24)$$

Introducing now (24) in (23) and performing the replacement $E - Q_0 - e_1 = m_2 x$ where $e_1 = \sqrt{m_1^2 + P^2}$ we obtain finally:

$$\mathcal{T}_0^{(1)} = \frac{\kappa_1}{VT} \frac{m_1 m_2}{P} \int_1^\infty \frac{1}{\sqrt{x^2 - 1}} \left(\frac{m_1}{e_1} x + 1 \right) \varphi_i \varphi_f^{(1)}. \quad (25)$$

In the above relation $\varphi_i, \varphi_f^{(1)}$ are the internal wave functions of the initial and final mesons and the integration limits have been deduced from the conditions $Q_z^2 \geq 0$ or $e_2 \geq m_2$ which make the square root real.

The expression of $\mathcal{T}_0^{(2)}$ can be obtained immediately by interchanging m_1 with m_2 and then the space-like electromagnetic form factor of pseudoscalar mesons reads:

$$f_{em}(t) = \frac{m_1 m_2}{M^2 V_0 L_0} \int_1^\infty \frac{dx}{\sqrt{x^2 - 1}} \left[\kappa_1 \left(\frac{m_1}{e_1} x + 1 \right) \varphi_i \varphi_f^{(1)} + \kappa_2 \left(\frac{m_2}{e_2} x + 1 \right) \varphi_i \varphi_f^{(2)} \right] \quad (26)$$

where $e_{1,2} = \sqrt{m_{1,2}^2 + P^2}$. We remark that V_0 does not really appear at the end because it will be cancelled out by a similar factor coming from the normalization condition of the internal wave functions. This is not the case for L_0 , which still remains in the expression of the electromagnetic form factor at $t \neq 0$. However, keeping in mind that the form factor is really normalized to unity at $t = 0$, we shall take L_0 as a free parameter enabling us to impose this constraint on the expression (26).

A first consequence of the relation (26), independent of the particular form of the internal wave functions is that $f_{em}^{\pi^0, \eta, \eta'}(t) \equiv 0$, while $f_{em}^{K^0, \bar{K}^0}(t) \sim (m_s - m_d)$ and in principle does not vanish.

III. NUMERICAL RESULTS

The numerical results we quote below have been obtained for

$$\varphi(p, q; Q) = \frac{\mathcal{N}}{(p \cdot q)^n} = \frac{2^n \mathcal{N}}{[(P - Q)^2 - m_1^2 - m_2^2]^n} \quad (27)$$

where \mathcal{N} is a normalization constant. With the notations in eq. (25) one has

$$\varphi_i \varphi_f^{(1)} = \frac{\mathcal{N}^2}{(m_1 m_2)^{2n} \left(x^2 + \frac{P^2}{m_1^2} \right)^n} \quad (28)$$

Observing that for $n = 1, 2$ the integral in the expression (26) can be performed analytically, we find:

$$f_{em}^{(1)}(t) = \frac{\mathcal{N}^2}{M^2 V_0 L_0} \sum_{i=1,2} \kappa_i \left(\frac{2\pi m_i^2}{4m_i^2 - t} + \frac{2m_i^2}{\sqrt{t^2 - 4m_i^2} t} \ln \frac{\sqrt{4m_i^2 - t} + \sqrt{-t}}{\sqrt{4m_i^2 - t} - \sqrt{-t}} \right) \quad (29)$$

$$f_{em}^{(2)}(t) = \frac{\mathcal{N}^2}{M^2 V_0 L_0} \sum_{i=1,2} \kappa_i \left[\frac{4\pi m_i^2}{(4m_i^2 - t)} + \frac{8m_i^2}{t^2 - 4m_i^2 t} + 4 \left(\frac{m_i^2}{\sqrt{t^2 - 4m_i^2} t} + \frac{m_i^4}{(4m_i^2 - t) \sqrt{t^2 - 4m_i^2} t} \right) \ln \frac{\sqrt{4m_i^2 - t} + \sqrt{-t}}{\sqrt{4m_i^2 - t} - \sqrt{-t}} \right], \quad (30)$$

which gives then:

$$f_{em}^{(1)}(0) = \frac{\mathcal{N}^2}{M^2 V_0 L_0} \left(\frac{\pi}{2} + 1 \right) \quad (31)$$

$$\left. \frac{df_{em}^{(1)}(t)}{dt} \right|_{t=0} = \frac{\mathcal{N}^2}{M^2 V_0 L_0} \sum_{i=1,2} \kappa_i \frac{1}{m_i^2} \left(\frac{\pi}{8} + \frac{1}{6} \right) \quad (32)$$

$$\lim_{t \rightarrow -\infty} f^{(1)}(t) \sim -\frac{1}{t} \ln(-t) \quad (33)$$

$$f_{em}^{(2)}(0) = \frac{\mathcal{N}^2}{M^2 V_0 L_0} \left(\frac{\pi}{4} + \frac{2}{3} \right) \quad (34)$$

$$\left. \frac{df_{em}^{(2)}(t)}{dt} \right|_{t=0} = \frac{\mathcal{N}^2}{M^2 V_0 L_0} \sum_i \kappa_{i=1,2} \frac{1}{m_i^2} \left(\frac{\pi}{8} + \frac{4}{15} \right) \quad (35)$$

$$\lim_{t \rightarrow -\infty} f^{(2)}(t) \sim \frac{1}{t^2} \ln(-t). \quad (36)$$

The comparison of the theoretical results with the experimental data has been done by looking for the values of the parameter n and of the quark masses giving the lowest possible χ^2 , where:

$$\chi^2 = \sum_i \frac{(f_{th}^2(t_i) - f_{exp}^2(t_i))^2}{\sigma_i^2}. \quad (37)$$

Here σ_i are the experimental errors and the sum is performed over all the experimental points quoted in [8]. The values we found for the pion and kaon electromagnetic form factors are given in Figs. 1 (a) and (b) respectively. They have been obtained for $n=1.5$ and the quark masses $m_{u,d}=430\text{MeV}$, $m_s=700\text{MeV}$. Our fit has $\chi_\pi^2=69.4$ for 45 experimental points and respectively $\chi_K^2=12.2$ for 15 experimental points, while the simple pole fit has $\chi_\pi^2=52.5$ and $\chi_K^2=12$. We also get $r_\pi^2=0.434\text{ fm}^2$, $r_{K^+}^2=0.333\text{ fm}^2$, $r_{K^0}^2=-0.069\text{ fm}^2$ while the experimental values are $(r_\pi^2)_{exp}=0.431\pm0.01\text{ fm}^2$, $(r_K^2)_{exp}=0.34\pm0.05$ and $(r_{K^0}^2)_{exp} = -0.054\pm0.026\text{ fm}^2$ [8].

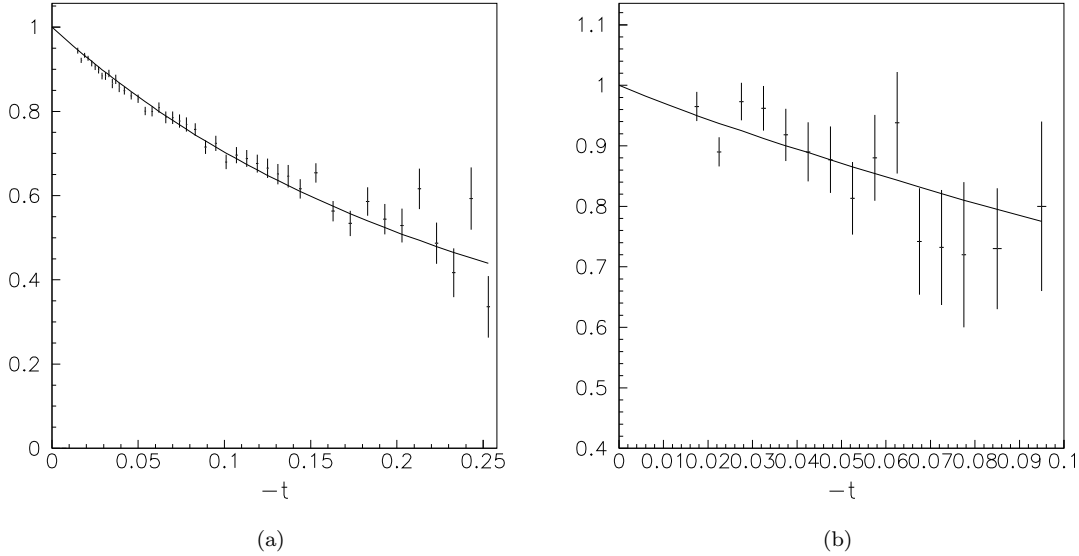


FIG. 1. Comparison of the theoretical predictions for the square of the pion (a) and kaon (b) electromagnetic form factors with data at low t (GeV^2/c^2).

We notice that the theoretical results are very sensitive with respect to the up and down quark masses. This let us believe that the numerical results quoted above may be improved by introducing a difference between the lightest quarks. We notice also that the values of the quark masses found from the best fit are a bit larger than the currently used constituent masses [1,6,9]. This is normal taking into account that the contribution of the transverse degrees of freedom has been included in the quark masses.

IV. COMMENTS AND CONCLUSIONS

The model we presented in this paper is a relativistic model for bound states. It enabled us to calculate the electromagnetic form factors by means of the overlap integral over the internal wave functions of the initial and final mesons.

The internal function we used is just a suitable trial function serving to illustrate the qualities of the model, not the solution of a dynamical scheme. We remark that in the infinite momentum limit it does not have any of the usual asymptotic forms quoted in [1,6]. This is because the distribution amplitude (27) writes in terms of real Lorentz scalar products which introduce "bad" components of momenta. It demonstrates, however, that the existence of the meson form factors can be explained through the stable, time invariant structure of the mesons and also, that the colour fluctuations contributing to the quark form factor can be neglected at low momentum transfer.

The remarkable point is that our results are unexpectedly consistent, as it can be seen from the comparison of the pion radius with the box length L_0 . First we mention that L_0 cannot be infinite or 0. In the first case $f_{em}(t) = 0$ for any $t \neq 0$ and mesons would behave like rigid bodies with respect to the electromagnetic interaction. In the second case mesons would be structureless.

In our specific case L_0 can be calculated from the normalization conditions of the meson state (5) and of the electromagnetic form factor (13). Freezing two spatial degrees of freedom, using the internal function (27) and performing the replacements $M - Q_0 = (m_1 + m_2)x$ and $Q_z = (m_1 + m_2)y$ we get:

$$\mathcal{J} = \frac{\mathcal{N}^2}{2MV_0} \frac{2^n}{(m_1 + m_2)^{2n-2}} \int_1^\infty dx \int_{-\zeta}^\zeta dy \frac{1}{[x^2 - y^2 - \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2}]^{2n}} \sqrt{\frac{x^2 - y^2 - 1}{x^2 - y^2 - \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2}}} = 1 \quad (38)$$

where $\zeta = \sqrt{x^2 - 1}$ and the integration limits are the consequence of reality of the norm.

Then, by using the values of the parameters giving the best fit, we obtain in the pion case $L_0 \approx 0.46$ fm, not far from the pion radius which is roughly 0.65 fm.

It is important to notice that the value we get for L_0 may help us to put an upper limit for the time T_0 involved in the definition of the effective field Φ . To this end we recall that L_0 is related with the overlapping time of the systems representing the mesons and conjecture that the model is valid up to meson energies where this time is sensibly larger than the time involved in the definition of the effective field Φ . This means $\frac{E L_0}{P} \gg \frac{E T_0}{M}$. Then, if we expect the model holds at least up to $t = -1$ GeV² we get $T_0 < 0.7 \cdot 10^{-25}$ s.

A last point we comment here concerns the continuity of the form factor from the space-like to the time-like region. In principle our expression (25) can be continued at $t > 0$, but it will have no significance in the absence of a dynamical scheme of the strong interaction.

Acknowledgements The author thanks Prof. H. Leutwyler for the kind hospitality at the Institute of Theoretical Physics of the University of Bern where this work was completed during author's visit supported by the Swiss National Science Foundation under Contract No. 7 IP 051219. The author thanks Fl. Stancu, University of Liège, for useful suggestions and careful reading of the manuscript. The partial support of the Romanian Academy through the Grant No. 329/1997 is also acknowledged.

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